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Tests of Significance for Temperature and Precipitation Normals

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ABSTRACT—A t-test is given to test monthly temperature normals, and a new test related to the F-test is developed for testing monthly precipitation normals. The theory involving the two errors committed in using such tests is discussed. This theory is used to set the probabilities of type 1 and 2 errors at predetermined levels. The tests are applied to examples from the normals revision program.

1. INTRODUCTION

In the early 1950s, the U.S. Weather Bureau computed a set of monthly temperature and precipitation normals based on the period 1921–50 according to newly adopted World Meteorological Organization "Technical Regulations." The latter also provided for the decennial review and revision of normals to provide unbiased estimates of normals for the 30-yr period ending every 10 yr. It therefore became necessary to review 1921–50 normals and to estimate normals for the period 1931–60. We define the normal to be the population mean; thus, all statistics we calculate will be statistical *estimates* of such normals. In accordance with common practice, we shall often refer to these estimates as normals. It should always be remembered, however, that we can never know the true normals or any other climatological parameters, but only statistical estimates of them. This is a basic concept of climatological analysis.

In the process of estimating the 1931–60 normals, we decided to test the difference between the 1931–60 and 1921–50 normals for statistical significance. This would not only provide a convenient tool in adjusting heterogeneous records, but would allow commercial users to retain 1921–50 normals where differences between the old and new normals are only random. In some commercial uses of temperature normals, the system involving them is rather complicated, and a change to new normals requires a costly modification of the system.

2. SIGNIFICANCE TEST FOR TEMPERATURE

As for the 1921–50 period, new normals were to be estimated for monthly average maximum temperature, monthly average minimum temperature, and the monthly average of these. Since all of these are close to normally distributed (Thom 1968) and require use of the difference method of adjustment, it is natural to test the simple difference between the mean temperatures for the two periods 1921–50 and 1931–60 by the t-test. The interval 1931–50 is common to both normal periods; hence, a test of the simple difference between the two periods would result in a test of the difference between the 1921–30 and 1951–60 periods. This would be all right if the 1921–30 period could always be separated, but, due to many adjustments for heterogeneity, this is not possible in most instances. Hence, it was necessary to test the mean for the 1951–60 period, the mean for the new data, against the old normal for the 1921–50 period. Clearly, this applies generally to any standard normal period.

We recall that Student's "t" is the ratio of the difference between the means divided by the standard error of that difference. The difference may be written

$$a=\bar{t}_{30}-\bar{t}_{10} \tag{1}$$

where $\bar{t}_{30}$ is the 1921–50 normal for a particular month, and $\bar{t}_{10}$ is the mean for 1951–60 for that month. Since the variance (or scale), v, of temperature is conservative for the heterogeneities in the records used in estimating normals,

$$\begin{aligned} v(\bar{t}_{30}-\bar{t}_{10})&=v(\bar{t}_{30})+v(\bar{t}_{10})\\ &=v(t)/30+v(t)/10\\ &=(2/15)v(t). \end{aligned} \tag{2}$$

The standard error of a is then the square root of eq (2), or

$$s(\bar{t}_{30}-\bar{t}_{10})=0.365\, s(t) \tag{3}$$

where $s(t)$ is the standard deviation of monthly average temperature. To avoid confusion of the two kinds of t's, we shall use w for Student's t. Thus, the test statistic is

$$w=\frac{(\bar{t}_{30}-\bar{t}_{10})}{0.365\, s(t)} \tag{4}$$

and w has the degrees of freedom of $s(t)$.

Since the correlation between average monthly maximum and average monthly minimum temperature is high but not, of course, one, their variances tend to average slightly higher than that for the average monthly temperature. Thus, it was deemed satisfactory to use the standard deviation of the average monthly temperature for testing the normals of average monthly maximum and average monthly minimum temperatures. This substitution gives slightly inflated w's for tests on the latter temperature normals.

3. PROPERTIES OF TESTS OF HYPOTHESIS

Although it is the usual practice in meteorological work to choose some standard significance limit probability such as 0.05 or 0.01, it is useful in most instances, and especially here, to give more detailed consideration to this choice. A statistical significance test or statistical test of hypothesis in its most basic sense consists of a hypothesis, commonly called the null hypothesis, a hypothesis alternative to this called the alternative hypothesis, and a rule based on probability of occurrence to decide which hypothesis to accept. The test statistic, the probability tables for the test statistic, and the significance limit or limits comprise the rule for deciding whether to accept or reject the null hypothesis. Rejecting the null hypothesis is equivalent to accepting the alternative hypothesis. The probability tables for a test are made from the distribution of the test statistic when the null hypothesis is true. In making a test, the null hypothesis is rejected (alternative hypothesis accepted) when the test statistic exceeds the significance limit or is outside the range over which the null hypothesis is to be accepted. The probability of being outside the range of acceptance of the null hypothesis when it is true is the significance limit probability (0.05, 0.01, etc.); hence, the probability of rejecting the null hypothesis when it is true is equal to the significance limit probability. This rejection of the null hypothesis when it is true or acceptance of the alternative hypothesis when it is false is, of course, an error and statisticians call it the type 1 error. The probability of committing this error in making a test of hypothesis is the significance limit probability.

Often the type 1 error is the only error considered; that is, the researcher chooses a significance limit and makes a test without giving consideration to the other error called the type 2 error. The type 2 error is acceptance of the null hypothesis when it is false, or what is equivalent, rejecting the alternative hypothesis when it is true. If the type 1 error is fixed, the type 2 error is reduced by controlling the trials artificially and increasing their number as can often be done in the laboratory. Thus, in physics experiments, the type 2 error is not ordinarily of as much concern. In meteorology, however, control is usually impossible and the number of trials is usually small and often impossible to increase. Hence, the type 2 error may be large and important to know. This is particularly true in testing new theory. For example, the type 2 error is rejection of the alternative hypothesis when it is true, and if the test does this with a high probability, a new hypothesis based on a desired alternative may be lost or not given proper consideration. One of us introduced these concepts in the evaluation of artificial augmentation of precipitation where cost ratio is very high, and one does not want to risk losing a large gain for a very low cost. If a small significance probability is employed, it is practically a certainty that the type 2 error will be large in most meteorological work as will be seen in two examples that follow.

We have seen that the probability of the type 1 error, α, is obtained from the distribution of the test statistic on the null hypothesis; that is, when the null hypothesis is true. In a like manner, the probability of the type 2 error, β, is obtained from the distribution of the test statistic on the alternative hypothesis; that is, when the alternative hypothesis is true. Since β is the probability of *rejecting* the alternative hypothesis when it is true, $1-\beta$ is the probability of *accepting* the alternative when it is true. This is called the power of the test since it measures the capability of the test to accept the alternative hypothesis, which is often the desirable outcome of an experiment. We are now in a position to discuss certain relationships between the various elements of a test.

For simplicity of explanation, we assume that the alternative hypothesis is greater than the null hypothesis; that is, we are testing whether the population value of the test statistic is the null value, H_0, or the alternative value, H_1, where $d=H_1-H_0$. The distribution of the test statistic when the null hypothesis is true will be spread about H_0 with the significance limit, b, to the right of H_0 on the test statistic scale. If a test statistic is to the left of b, the null hypothesis will be accepted; if to the right of b, the alternative hypothesis will be accepted. The area under the null distribution to the right of b is α or the significance limit probability. The distribution of the test statistic when the alternative hypothesis is true is spread around H_1, which is to the right of H_0. The probability of a type 2 error, β, is then the area of the alternative distribution to the left of b.

The effect of variation in the test parameters: the type 1 error probability, α (or significance limit, b), the sample size, n, and the size of the alternative, H_1 (or its distance d from the null value, H_0), on the type 2 error may now be seen. For fixed sample size and alternative, H_1, if α is made smaller, b becomes larger, and there is larger area β of the alternative distribution to the left of b. If α is made larger, b moves to the left and the area to the left of b is less so β is smaller. Thus, the probability of the type 2 error varies inversely with α. This is important in meteorological trials, for decreasing α also decreases the ability to accept the alternative. Thus, if the alternative is an improved method or a potentially valuable new theory, it will often be rejected with high probability if the significance probability α is made too small. It is for this reason that we have frequently adopted a basic value of $\alpha=0.10$ for meteorological work where nothing definite is known about the power of the test. If the type 2 error can be determined, of course, the significance limit can be given more detailed consideration as will be seen below.

For fixed α and H_1, it is seen that as H_1 moves closer to H_0, more area on the left tail of the alternative distribution is cut off by the significance limit, thus increasing the type 2 error or decreasing the power. As H_1 moves farther to the right away from H_0, less area is cut off and the type 2 error is decreased or the power is increased. This fits with our intuitive notion since, if H_0 and H_1 are close together, it is more difficult to differentiate between them than when they are farther apart.

For fixed α and H_1 one can see that if n is increased, the dispersion or spread of both the null and alternative distributions decreases because the sampling variation of the test statistic decreases with sample size increase. This not only causes the significance limit to move toward the left, cutting off less area on the alternative distribution, but the decrease in spread of the alternative distribution also causes less area to be cut off on b, and both cause a decrease in type 2 error or an increase in power. Again, this fits our intuitive reasoning, for the more trials we have the more sure we are of correctly accepting or rejecting a given hypothesis.

4. RISKS OF ERROR FOR TEMPERATURE TESTS

We have seen that α, β, d, and n are the parameters of the classical test of hypothesis we are concerned with here. Since n is always fixed before a sample is drawn, with a given alternative only α and β can vary. These are probabilities and hence are determined by frequency distributions. In the case of Student's test used for temperature normals, α is determined by the null distribution, which is Student's distribution or the t-distribution (w-distribution here). This is located centrally around the null hypothesis, H_0. β is determined by a more complicated distribution called the noncentral t-distribution and located centrally about the alternative hypothesis, H_1, noncentral to the null distribution.

Tables of Student's distribution are, of course, readily available, and, although several tables of noncentral t are available, they are rather complicated to use. Fortunately, there is a good approximation that relates α, β, d, and degrees of freedom, f, that requires use of only normal probability tables. For the two-sided test, this is

$$u_{1-\beta}=\frac{f(|d|-u_{1-\alpha/2})-1.21\ u_{1-\alpha/2}(u_{1-\alpha/2}-1.06)}{f+1.21\ (u_{1-\alpha/2}-1.06)} \tag{5}$$

where $u_{1-\beta}$ is a unit normal quantile. The two-sided test is one for which the alternative hypothesis is both on the right and left of the null hypothesis; that is, one which tests for the true value being above or below the null hypothesis. We must use this test because we wish to test for both positive and negative departures of the new normals. To use eq (5) for one-sided tests, we merely replace $\alpha/2$ with α.

When methods for the preparation of the 1931–60 normals were first being considered, it was thought that a statistical test would be used to decide whether a normal was to be revised or left standing. Later, in view of international recommended practices, it was decided to estimate new normals regardless of the result of the significance test although the test was used in helping to determine heterogeneities. The test was also to be used to determine the significance of, and therefore the reality of, the difference between 1921–50 and 1931–60 normals so that those that showed significance could be marked. This provided industrial users with a means of deciding whether or not they must change a particular month's normal in their computational procedures. Under future conditions, it is recommended that only those normals be changed that are significantly different from the last normals. Our discussion now relates to the 1941–70 period.

To decide upon a reasonable alternative hypothesis, we employed the following reasoning: the maximum standard deviation of monthly average temperature over the United States from Thom (1968) is about 11°F in winter and 4°F in summer. The maximum standard errors in winter and summer for a 30-yr mean are therefore $1/\sqrt{30}$ of these values or about 2.0° and 0.7°F. We would, therefore, like to have our significance test find real departures of 0.7 of a standard error or more due to the use of the previous normal period. A departure of $|0.7|$ on the unit normal distribution (mean of zero and standard deviation of one) has a probability of 0.50 of being exceeded. The general value of d for a particular set of record periods is

$$d=\bar{t}_{31-60}-\bar{t}_{61-70}. \tag{6}$$

Also, the required new normal is

$$t_{41-70}=\frac{2}{3}\,\bar{t}_{41-60}+\frac{1}{3}\,\bar{t}_{61-70}. \tag{7}$$

Substituting, we get

$$\bar{t}_{41-70}=\frac{2}{3}\,\bar{t}_{41-60}+\frac{1}{3}\,\bar{t}_{31-60}-\frac{1}{3}\,d. \tag{8}$$

It is seen that d has the weight one-third, therefore, we will wish to recognize alternatives of $d/3=0.7$ or more or $d=2.1$°F. These departures can be either positive or negative so that the test must be a two-sided test with $|d|=2.1$°F.

Although more complicated weightings would result in risk functions relating the two types of errors, we felt that not enough information was available for such an analysis. Consequently, we have assumed that it would be as bad to revise a normal when it did not need it as it would be not to revise one when it really did need it. Hence, we wish to make the probability of a type 1 error equal to the probability of a type 2 error with alternative $|d|$ equal to 2.1°F.

For the two-sided test, the alternative is $H_1-H_0=|d|$. Unfortunately, even though the type 1 and type 2 error probabilities are equal, $u_{1-\alpha/2}\neq u_{1-\beta}$; so eq (5) can only be solved for the required conditions by approximation. For one-sided tests, $u_{1-\alpha}=u_{1-\beta}$, hence eq (5) can be put in quadratic form and the proper root, $u_{1-\alpha}$, easily found for a given value of d. Solving eq (5) for d we have

$$|d|=(u_{1-\alpha/2}+u_{1-\beta})\left[1+\frac{1.21}{f}\,(u_{1-\alpha/2}-1.06)\right]. \tag{9}$$

After several trials, we found that for $\alpha=\beta=0.20$, and

$f=29$ degrees of freedom,

$$|d|=(1.282+0.842)\left[1+\frac{1.21}{29}(1.282-1.06)\right]=2.1\cdot$$

Thus, for the 0.20 significance limit or 0.20 probability of a type 1 error, the probability of a type 2 error is also 0.20. The power of the test is $1-0.20=0.80$. This means that the test gives a correct decision to revise a normal in 0.80 of the tests based on a departure $|d|$ having an additive effect of ± 0.7 of a standard error in determining the 30-yr mean.

This is a very satisfactory power when compared to the many situations where no consideration has been given to the power. For example, with the same alternative and record length $n=30$ for a 0.05 significance limit, $u_{1-\alpha/2}=u_{1-0.025}=1.960$, eq (5) now gives

$$u_{1-\beta}=\frac{29(2.1-1.96)-1.21(1.96)(1.96-1.06)}{29+1.21(1.96-1.06)}=0.07.$$

From normal tables we find $1-\beta=0.53$. Thus, the power is only 0.53 with a 0.05 significance limit. For the 0.01 significance limit, the power is only 0.37. This means that the alternative hypothesis will only be accepted a little over one-third of the time when it is true. Thus, neither the 0.05 or 0.01 significance limits give satisfactory powers.

5. SIGNIFICANCE TEST FOR PRECIPITATION

The problem of testing precipitation normals is similar to that for temperature; that is, it is desired to test whether the substitution of a subsequent 10-yr period of record would make a significant difference in the 30-yr normal. The adjustment of precipitation means and totals is done by the ratio method; hence, it is natural to use a ratio as the test statistic. Barger and Thom (1949) and Thom (1958) showed that monthly precipitation is distributed in a gamma distribution. Since the totals or averages of several individual gamma variates with the same scale again have gamma distributions, we will be concerned with the ratio of two gamma variates. It is well known that this ratio has a beta distribution of the second kind. This is also the familiar F-distribution with parameters of the beta distribution equal to $f_1/2$ and $f_2/2$ where f_1 and f_2 are the degrees of freedom in the two variances whose ratio is F. Since we are not concerned with degrees of freedom here, and since the beta parameters need not be integers, the beta distribution that we need to use is more general than the F-distribution.

We take the test statistic to be the ratio of the total for the 10-yr record for a particular month for the period 1961–70 to the total for that month for the normal period 1931–60. If we express the 10-yr total as $x_1=\Sigma p$ and the 30-yr total as $x_2=30\overline{p}_{30}$, where the p's are values for the individual months during the 10-yr period, and $\overline{p}_{30}$ is the normal for the 1931–60 period, we may express the test statistic as

$$q=x_1/x_2. \tag{10}$$

The distribution of the total of 10 identical gamma variates with parameter γ is a gamma distribution with parameter 10γ. Likewise, $30\overline{p}_{30}$ is distributed in a gamma distribution with parameter 30γ. Thus, q has a beta distribution with parameters 10γ and 30γ.

While the beta distribution is well known, there were no tables for beta that cover the range of γ required; therefore, a new table had to be prepared. If the parameter of the precipitation distribution for a given month is γ and the parameters of the beta distribution of q are γ_1 and γ_2, then $\gamma_1=10\gamma$ and $\gamma_2=30\gamma$. Since γ_2 is always three times γ_1, our beta table need only involve γ_1; in fact, it can be prepared for 10γ so that all that is necessary to use the table is to multiply the γ for the monthly precipitation distribution by 10 and enter the table to obtain the limits of probability. Before we proceed to developing a table for testing the significance of q, we need to settle on the significance probability levels or the type 1 error, α. As we have seen above, α will be related to β, the type 2 error, and inasmuch as we wish to control both errors, we need to study the relation between α, β, and the alternative hypothesis.

According to the principles of testing hypotheses given above, we will accept the null hypothesis when the probability on q is

$$P\left[B^{-1}\left(\frac{\alpha}{2};10,30\right)\leq q\leq B^{-1}\left(1-\frac{\alpha}{2};10,30\right)\right]=1-\alpha. \tag{11}$$

$B^{-1}(\alpha/2;10,30)$ is the inverse of the beta distribution with parameters 10 and 30; in other words, it is a quantile of the B distribution. For simplicity we drop the parameters and simply write $B^{-1}(\alpha/2)$ for the quantile. To obtain the type 2 error, β, we write eq (11) conditional on the value of the alternative as follows:

$$P\left[B^{-1}\left(\frac{\alpha}{2}\right)\leq q\leq B^{-1}\left(1-\frac{\alpha}{2}\right)\bigg|\varphi=\xi_1/\xi_2\right]\cdot \tag{12}$$

Dividing the inequality by $\varphi=\xi_1/\xi_2=\varphi$ gives

$$P\left[\frac{B^{-1}\left(\frac{\alpha}{2}\right)}{\varphi}\leq\frac{x_1/\xi_1}{x_2/\xi_2}\leq\frac{B^{-1}\left(1-\frac{\alpha}{2}\right)}{\varphi}\right]=\beta.$$

Thus, the beta variate x_1/x_2 is subject to a single scale change which is again a beta variate. Hence,

$$P\left[\frac{B^{-1}\left(\frac{\alpha}{2}\right)}{\varphi}\leq q\frac{\xi_2}{\xi_1}\leq\frac{B^{-1}\left(1-\frac{\alpha}{2}\right)}{\varphi}\right]=\beta. \tag{13}$$

It remains to evaluate $B^{-1}(\alpha/2)$ and $B^{-1}(1-2/\alpha)$ for the significance test of q and the evaluation of β for a reasonable alternative.

Cochran (1940) has given approximations to Fishers' distribution of z that are related to our variable, q. The general formula for all probabilities is

$$z=\frac{u}{\sqrt{(h-\lambda)}}-\left(\frac{u^2+2}{6}\right)\left(\frac{1}{f_1}-\frac{1}{f_2}\right) \tag{14}$$

where u is the unit normal deviate, f_1 and f_2 are degrees of freedom in the z distribution, λ is a function of the probability associated with u, and h is given by

$$\frac{2}{h}=\frac{1}{f_1}+\frac{1}{f_2}. \tag{15}$$

The β, F, and z variates are related by

$$\frac{f_2 q}{f_1}=F=e^{2z},$$

which after taking logarithms gives

$$\ln q+\ln\left(\frac{f_2}{f_1}\right)=2z. \tag{16}$$

Since the beta distribution parameters are $f_1/2$ and $f_2/2$, $f_1=2\gamma_1$, and $f_2=2\gamma_2$. Also, as has been seen previously, $\gamma_2=3\gamma_1$; hence,

$$\frac{f_2}{f_1}=3. \tag{17}$$

Substitution in the last factor of eq (14) and in eq (15) yields

$$\frac{1}{f_1}-\frac{1}{f_2}=\frac{1}{3\gamma_1} \tag{18}$$

and

$$h=3\gamma_1. \tag{19}$$

Substitution of eq (17) in eq (16) yields

$$\ln q+\ln 3=2z. \tag{20}$$

Substitution of eq (18), (19), and (20) in eq (14) gives

$$\ln q=\frac{2u}{\sqrt{(3\gamma_1-\lambda)}}-\frac{u^2+2}{9\gamma_1}-\ln 3. \tag{21}$$

Raising to the exponential gives the computational form

$$q=\frac{1}{3}\exp\left[\frac{2u}{\sqrt{(3\gamma_1-\lambda}}-\frac{u^2+2}{9\gamma_1}\right]. \tag{22}$$

Cochran (1940) gives the following formula for λ as a function of u:

$$\lambda=\frac{(u^2+3)}{6}. \tag{23}$$

This gives the value of q for $\gamma_1=10\gamma$ where γ is the shape parameter of the precipitation distribution for a given month.

Equation (22) will be used in approximating β. Also needed is the inversion of eq (21), which is the quadratic

$$u^2-\frac{18\gamma u}{\sqrt{(3\gamma_1-\lambda)}}+9\gamma_1 \ln 3q+2=0$$

for which the pertinent root is

$$u=\left\{\frac{9\gamma_1}{\sqrt{(3\gamma_1-\lambda)}}-\left[\frac{(9\gamma_1)^2}{3\gamma_1-\lambda}-9\gamma_1\ln 3q-2\right]^{1/2}\right\}. \tag{24}$$

From eq (13), we see that to calculate the type 2 error probability, we must assume a value of φ which gives a desired alternative. If we choose $\varphi=1.5$, the alternative is $(1/3)(2/3)=2/9$. Thus, the alternative is 1/9 less than the mean value of q which is close to 1/3. After approximating α and β several times for (an average value)$\gamma_1=40$ from eq (22) and (24), we found, using $\alpha/2=0.10(u-1.282)$ in eq (22) with Cochran's value for λ, that

$$\ln q_{0.10}=\frac{-2.564}{\sqrt{(120-0.77)}}-\frac{3.654}{360}-1.09861=-1.34377$$

and

$$q_{0.10}=\exp(-1.34377)=0.261.$$

Similarly,

$$\ln q_{0.90}=0.23501-0.01015-1.09861=-0.87375$$

and

$$q_{0.90}=\exp(-0.87375)=0.417.$$

Thus, the null hypothesis will be rejected if a sample, q, lies outside the interval (0.261, 0.417) and accepted if it is in this interval.

Applying eq (13) with $\varphi=1.5$, we find

$$\beta=P(0.666\times 0.261<q<0.666\times 0.417)$$
$$=P(0.174<q<0.278).$$

This is equivalent to

$$\beta=P_2(q<0.278)-P_1(q<0.174). \tag{25}$$

We may employ eq (24) to evaluate the P's. Substituting the limits from eq (25) gives $u_1=-3.33$ and $u_2=-0.95$. Since the u's are unit normal deviates, from normal distribution tables we find $N(-3.33)=0.00043$ and $N(-0.95)=0.17106$. Hence, by eq (25),

$$\beta=P_2-P_1=0.17106-0.00043=0.17.$$

If φ is changed slightly to 1.47, we find $u_1=-3.25$ and $u_2=-0.84$ for which $N(-3.25)=0.00$ and $N(-0.84)=0.20$ from which β is now almost exactly 0.20. Thus, for an average value of γ_1 of 40, a type 1 error probability of 0.20, and an alternative that is 1/9 less than the mean value of q, the probability of a type 2 error is very nearly equal to the probability of a type 1 error. Similar results are obtained for an alternative above the mean value of q. The test then meets the requirements set out above and appears to be satisfactory for precipitation.

The beta distribution (table 1) was prepared for $\alpha=0.20$ as indicated above with parameters 10γ and 30γ, or simply 10γ. For 10γ from 1 to 16, Pearson's tables (1934) were used. Above $10\gamma=16$, eq (22) was employed. Cochran's (1940) results indicate that the table should be reliable in the last decimal place.

6. APPLICATION OF THE TESTS

We apply the tests to two stations to illustrate the simple computation. From official temperature records (U.S.

TABLE 1.—*Beta distribution for $\alpha=0.20$*

10γ	$q_{0.10}$	$q_{0.90}$	10γ	$q_{0.10}$	$q_{0.90}$
1	0. 040	1. 239	51	0. 269	0. 407
2	. 089	. 835	52	. 269	. 406
3	. 119	. 712	53	. 270	. 406
4	. 140	. 647	54	. 270	. 405
5	. 156	. 607	55	. 271	. 404
6	. 168	. 578	56	. 271	. 403
7	. 178	. 556	57	. 272	. 403
8	. 186	. 539	58	. 272	. 402
9	. 193	. 526	59	. 273	. 402
10	. 199	. 514	60	. 273	. 401
11	. 205	. 505	61	. 274	. 400
12	. 209	. 496	62	. 274	. 400
13	. 213	. 489	63	. 275	. 399
14	. 217	. 483	64	. 275	. 399
15	. 221	. 477	65	. 276	. 398
16	. 224	. 472	66	. 276	. 398
17	. 227	. 467	67	. 276	. 397
18	. 229	. 463	68	. 277	. 397
19	. 232	. 459	69	. 277	. 396
20	. 234	. 456	70	. 278	. 396
21	. 236	. 453	71	. 278	. 395
22	. 238	. 449	72	. 278	. 395
23	. 240	. 447	73	. 279	. 394
24	. 242	. 444	74	. 279	. 394
25	. 244	. 442	75	. 279	. 393
26	. 245	. 439	76	. 280	. 393
27	. 247	. 437	77	. 280	. 393
28	. 248	. 435	78	. 280	. 392
29	. 249	. 433	79	. 281	. 392
30	. 251	. 431	80	. 281	. 391
31	. 252	. 430	81	. 281	. 391
32	. 253	. 428	82	. 282	. 391
33	. 254	. 426	83	. 282	. 390
34	. 255	. 425	84	. 282	. 390
35	. 256	. 424	85	. 282	. 390
36	. 257	. 422	86	. 283	. 389
37	. 258	. 421	87	. 283	. 389
38	. 259	. 420	88	. 283	. 389
39	. 260	. 418	89	. 284	. 388
40	. 261	. 417	90	. 284	. 388
41	. 262	. 416	91	. 284	. 388
42	. 263	. 415	92	. 284	. 387
43	. 263	. 414	93	. 285	. 387
44	. 264	. 413	94	. 285	. 387
45	. 265	. 412	95	. 285	. 386
46	. 265	. 411	96	. 285	. 386
47	. 266	. 410	97	. 286	. 386
48	. 267	. 409	98	. 286	. 386
49	. 267	. 409	99	. 286	. 385
50	. 268	. 408	100	. 286	. 385

Department of Commerce 1956), we find the 1921–50 normal average maximum temperature for March for Boise, Idaho, to be $\bar{t}_{30}=52.2°$F. From published records, we find the average maximum temperatures for March 1951–60, which when averaged gives $\bar{t}_{10}=50.4°$F for the 10-yr mean. From Thom (1968), we read the standard deviation for March at Boise to be 2.9°F. Substituting these values in eq (4), we find

$$w=(52.2-50.4)/(0.365\times 2.9)=1.70.$$

Since this is greater than 1.31, the upper 0.10 value of w, or outside the range ± 1.31, the 10-yr mean is significantly smaller than the old normal. This indicates that the 1931–50 record should be adjusted to a new regime indicated by the 1951–60 record. If the value had been in the range $w=\pm 1.31$, we would have decided that there was no significant change in the record between 1931 and 1960.

To apply the test for precipitation to August precipitation for Hartford, Conn., we again used the U.S. Department of Commerce (1956) data. The 1921–50 normal precipitation for August was $\bar{p}_{30}=3.54$ in. From published records, we obtained the 10 individual August totals for which the sum is 50.62 in. Using eq (10), we find

$$q=\frac{50.62}{30\times 3.54}=0.477.$$

From Thom and Vestal (1968) we found $10g$, a statistical estimate of $\gamma_1=10\gamma$, to be 48. Referring to the beta distribution table, we find $q=0.409$ for the 0.90 value at $10\gamma=48$. Thus, the q for August is greater than the 0.90 value 0.409 or outside the interval (0.267, 0.409) and is therefore significant. This necessitated an adjustment of the 1931–50 record to the 1951–60 period. If the value of q had fallen on the interval (0.267, 0.409), it would not have been significant, and we would have decided against any adjustment.

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PICTURE OF THE MONTH
Severe Weather Situation, March 28, 1972

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Tornadoes, funnel clouds, and hail were reported throughout the Southern States on Mar. 28, 1972. This outbreak of severe weather developed from two separate systems: a mature system in the southeastern States and a system that developed in eastern Texas.

The early morning Applications Technology Satellite 3 (ATS 3) view (fig. 1) showed a large area of convective cloudiness (C) stretching from Georgia southwestward along the gulf coast. This cloudiness was associated with the rapidly moving midtropospheric Low that had produced severe weather in Louisiana the day before and was now centered over Alabama. Just prior to this picture, two tornadoes formed in the trailing edge of this cloud system: one occurred northeast of Panama City, Fla., at 1405 GMT and one touched down near Marianna, Fla., at 1445 GMT. Figure 2 shows this area more than 4 hr later (1933 GMT). The main cloud system is now located along the east coast and is most active across northern Florida into the Gulf of Mexico (D–E). A tornado was reported west of St. Augustine at this time. No additional severe weather was reported, and the convective activity gradually diminished over Florida as the main weather system moved offshore.

Early in the day (fig. 1), low clouds covered a large area from Louisiana northwestward into Colorado (L–M). By 1933 GMT (fig. 2), the stratiform clouds in Louisiana and Texas had dissipated leaving fair weather cumulus over the region of moist southwesterly flow (G). Most of Texas remained clear until 1904 GMT. At that time, a faint line of towering cumulus became visible just west of Houston. These clouds marked the convergence zone between the warm moist Gulf air and the drier continental

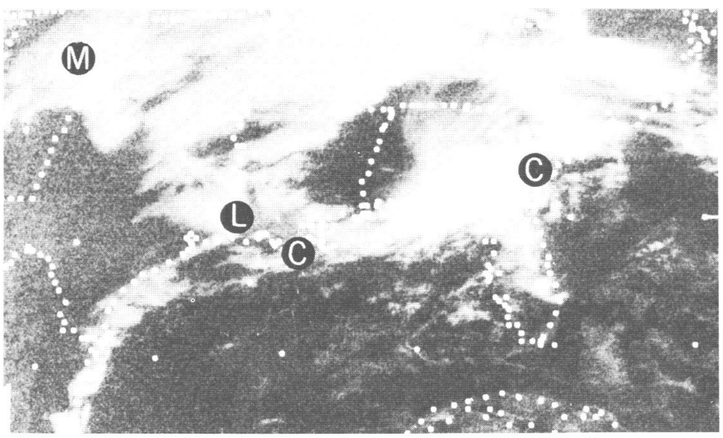

FIGURE 1.—ATS 3 photograph, Mar. 28, 1972, 1454 GMT.

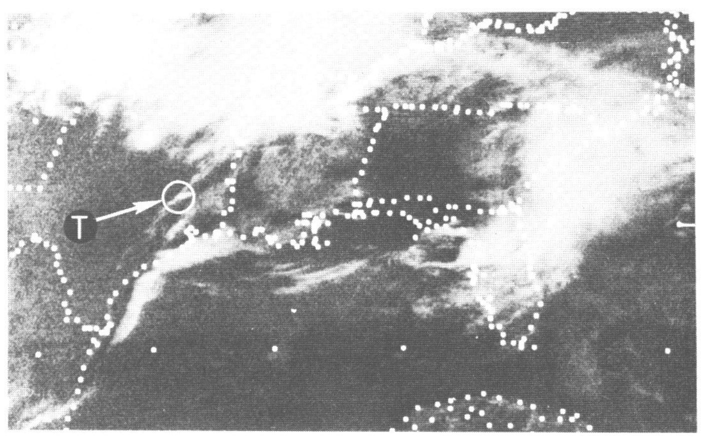

FIGURE 3.—Same as figure 1 except 2011 GMT.

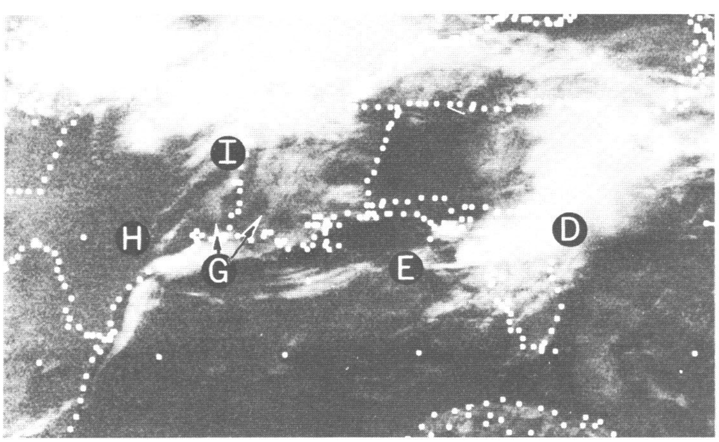

FIGURE 2.—Same as figure 1 except 1933 GMT.

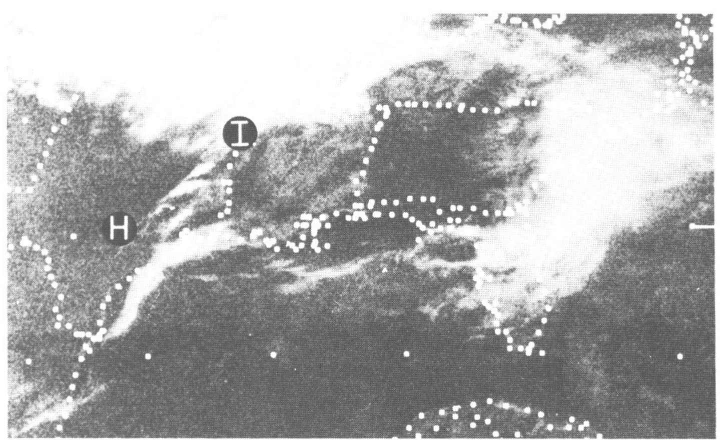

FIGURE 4.—Same as figure 1 except 2038 GMT.

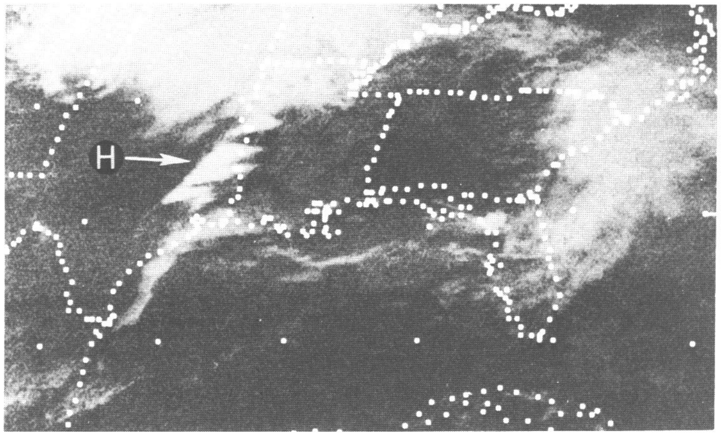

FIGURE 5.—Same as figure 1 except 2131 GMT.

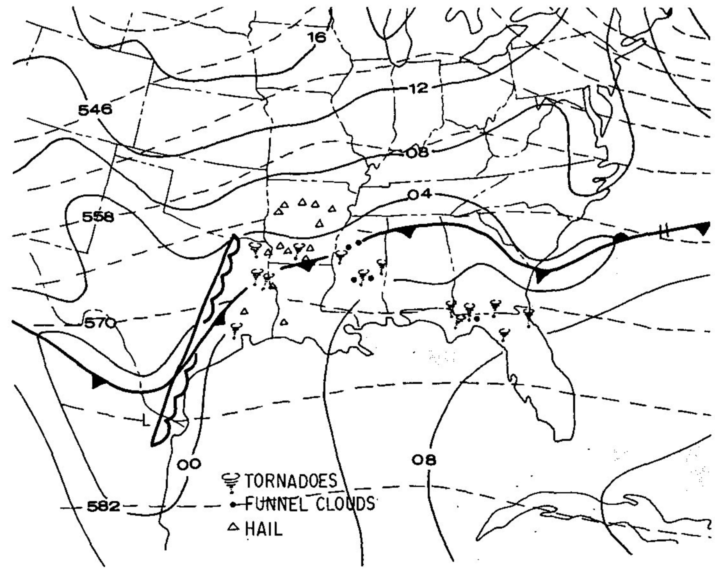

FIGURE 6.—Surface isobars (solid lines) and dew-point front at 2100 GMT, 500-mb contours (dashed lines) at 1200 GMT, and severe weather, Mar. 28, 1972.

air. The clouds that mark this "dry-line" can be seen stretching from H to I in figure 2. This line-feature has the characteristics of a potential tornado-producing system. By 2011 GMT (38 min later) (fig. 3), two distinct thunderstorm complexes (T) with anvil tops extended eastward from this line. Figure 4, taken 27 min later, shows further development of these thunderstorms along the dry-line (H–I). Radar reports indicated this to be an area of rain showers at 1945 GMT that became a line of thunderstorms with tops to 46,000 ft. by 2045 GMT. Figure 5 was taken as the first tornado spawned by this system touched down at Henderson, Tex. (H). Hail measuring 9 in. in diameter fell from the northernmost thunderstorm cluster near Texarkana, Tex., at 2310 GMT. This squall line moved eastward, producing eight tornadoes, three funnel clouds, and numerous hailstorms throughout Texas, Arkansas, Louisiana, and Mississippi (fig. 6).

Recent studies of frequent-interval pictures from ATS 3 have allowed meteorologists to recognize the potentially severe cloud patterns shown here. Satellite data often precede radar data in locating lines of developing cumulonimbus. These data are presently being acquired and incorporated into the operations and forecasts of the National Severe Storm Forecast Center in Kansas City, Mo.